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PREDICTION OF A FINITE BARE ELECTRICAL CHARGE FROM QUANTUM GRAVITY

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Abstract

In the spirit of general relativity, spacetime should become curved due to the presence of a particle of a given mass and charge. We try to understand this fact in the quantum theory of a thin shell of matter. It leads to a generalization of the potential energy of the shell in the semiclassical highest dominant order of Planck mass. Rather surprisingly, the quantization of charge is obtained as a consequence of the existence of bound states and the quantum of bare electrical charge is simply $e^2 = \hbar c/2$.

Pure particle states could be defined by the the total rest mass and interaction energies which also include self energies. In the classical point electron theory it was not possible, however, to obtain a finite model such that the total mass arises from the coupling to the electromagnetic field. This is because the self energy diverges linearly with the scale length, a, without any possible compensation,

$$mc^2 \sim e^2/2a. \tag{1}$$

Of course, gravity should enter into more realistic discussions and one would expect that its negative self energy would lead to a finite compensation for Eq. (1) at some scales. In fact, the picture of a simple spherical shell of charge can not be mantained but, instead, one should think on a particle as a field that curves spacetime due to its own self energy. A major purpose of theoretical physics is to make a mathematical model for a physical problem and it seems reasonable to start by making simple assumptions. In the present case, the simplest model is the classical dynamics of a spherical thin shell of mass with a given electric or magnetic charge embeded in a four dimensional spacetime. The problem has known solutions since it was earlier solved by Israel[1] who showed that the relevant physical description is much better approximated by a non static situation corresponding to the dynamics of the radius a of the shell:

$$M = -a\{(1 - 2M/a + Q^2/a^2 + \dot{a}^2)^{1/2} - (1 + \dot{a}^2)^{1/2}\}, \tag{2}$$

where, $M = Gm/c^2$, $Q^2 = Ge^2/c^4$ define the mass and the charge of the thin shell and derivatives are taken with respect to the proper time; the geometry of the shell corresponds to that of a three dimensional spacetime section embeded in the extended four dimensional one. The normal (timelike) coordinate to the shell is the Gaussian time of an exterior

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Reissner-Nordstrom geometry. Thus, Eq. (2) should be equivalent to the the full set of Einstein equations and the picture is rather analogous to that of the proper motion of a bubble of matter inmersed in its own curved spacetime. One can also write the equation of motion in a simpler, suggestive form

$$M = M(1+\dot{a}^2)^{1/2} + \frac{Q^2 - M^2}{2a}, \tag{3}$$

which represents the total dynamical energy in terms of self interations and the kinetic energy. It may be visualized as the effective hamiltonian constraint:

$$H(a,\dot{a}) \equiv M - M(1+\dot{a}^2)^{1/2} - \frac{Q^2 - M^2}{2a} = 0, \tag{4}$$

which should more properly be expressed in terms of the canonical momentum,

$$P \equiv \int \frac{\partial H(a,\dot{a})}{\partial \dot{a}} \frac{d\dot{a}}{\dot{a}} = -M sh^{-1}(\dot{a}) + G(a). \tag{5}$$

$G(a)$ so far an arbitrary function which does not change the classical equation of motion. Feeding it into the Hamiltonian constraint we get,

$$H(a,P) \equiv M - \frac{Q^2 - M^2}{2a} - M ch\left(\frac{P - G(a)}{M}\right) = 0. \tag{6}$$

It approximately corresponds to the Hamitonian derived by Berezin et al.[2], however, the ADM and shell masses were treated by Berezin as independent variables; on the other hand, we are here concerned on the case of charged particles rather than in black holes and, correspondingly, our analysis will lead to different and independent conclusions. Thus, $Q^2 > M^2$ (but notice that such a condition does not corresponds to the existence of naked singulaties because they are absent, by construction, in the thin shell model).

The standard quantization procedure consists on replacing a and P by quantum operators satisfying Dirac commutation relation acting on the wave functional of the dynamical variable (L_{Pl}^2 stands for Planck length squared)

$$[\hat{P}, \hat{a}] = -i L_{Pl}^2, \tag{7}$$

thus, the Hamiltonian constraint leads (modulo factor ordering) to the Wheeler-deWitt equation:

$$\{M - \frac{Q^2 - M^2}{2\hat{a}} - M ch\left(\frac{\hat{P} - G(\hat{a})}{M}\right)\}\Psi = 0. \tag{8}$$

According to the principle of correspondence one would expect the quantum theory to be approximated by semiclassical solutions of the form:

$$\Psi \sim K(a) \exp\left[\frac{i \int P(a) da}{L_{Pl}^2} + O(L_{Pl}^2)\right], \tag{9}$$

where $K(a)$ is a convenient Pauli-Van Vleck-Morette prefactor depending on the factor ordering and $P(a)$ is obtained from the classical constraint (hereafter we will write low case letters $P/L_{Pl}^2 \to p/\hbar$ when using standard units):

$$p(a) = g(a) + mc[ch^{-1}(1 + \frac{\alpha}{2mc^2 a/e^2})], \tag{10}$$

$\alpha \equiv Gm^2/e^2 - 1$. Now, Eq. (6) states that the total energy of the shell vanishes and it may also be contemplated as a zero energy problem of motion within the action of an effective potential energy defining the classical allowed values of the shell radius,

$$p^2 \equiv -mV(a), \tag{11}$$

that is,

$$V(a) = -mc^2[\frac{g(a)}{mc} + ch^{-1}(1 + \frac{\alpha}{2mc^2a/e^2})]^2. \tag{12}$$

On the other hand, $g(a) = 0$ when $x = mc^2a/e^2 \gg 1$ for in that case one should recover the usual post-Newtonian expressions (we use $ch^{-1}(1 + \alpha/2x) \sim (\alpha/x)^{1/2}$ for $x \gg 1$),

$$V(a) \sim \frac{e^2}{a} - \frac{Gm^2}{a}. \tag{13}$$

Notice that the effective potential energy of the shell has been obtained for the solutions of Einstein-Maxwell equations and it should be viewed as an exact classical result for the shell dynamics.

For $e^2 > Gm^2$ (i.e., $\alpha < 0$) and $x = mc^2a/e^2 < |\alpha|/4$, $V(a)$ would become a complex number unless $g(a) = -mci\pi$ and it should be the right selection because the effective potential should remain real for all values of the radius of the shell. These sort of results have to be expected in the semiclassical analysis. It motivates the definition:

$$V(x) = -mc^2[ch^{-1}(\frac{|\alpha|}{2x} - 1)]^2 \sim -mc^2[\log(\frac{|\alpha|}{x} - 2)]^2, \tag{14}$$

for $x < |\alpha|/4$. This is a very surprising result: even neglecting the action of classical gravity (i.e., when we formally put $G \to 0$), an atractive region arises in the semiclassical limit. There is no alternative classical analogous to this phenomenon. Hence, it seems to allow the complection of the old classical program of obtaining a finite value of the self energy of the electron. On the other hand, quantum-mechanically, severe restrictions have to be imposed to the (so far arbitrary) parameters of the potential in order that there were bound states:

$$2\int_0^{|\alpha|[(e^2/mc^2)-Gm^2/c^2]/4} p(a)da \geq \hbar/2, \tag{15}$$

the ground state satisfies the equality, that is (using Eq. (11)),

$$2mc\int_0^{|\alpha|/4} ch^{-1}(\frac{|\alpha|}{2x} - 1)d[\frac{e^2}{mc^2}x] = \frac{\hbar}{2}, \tag{16}$$

and,

$$e^2 = Gm^2 + \frac{\hbar c}{2}. \tag{17}$$

Therefore, in the limit of negligible gravitational self energy, the ground state is defined by $e^2 = \hbar c/2$. It represents the quantization of the electric charge. Of course we must consider $\hbar c/2$ as the quantum of the bare charge. We conclude that quantum gravity should induce a finite value of the fine structure constant to high energies.

The above result, obtained using a very simplified exact solvable model, should be correct to the highest order in the semiclassical expansion for the solution of the full quantum gravity Wheeler-deWitt equation. It is a merit of the semiclassical program of quantization that this result takes place. It also amounts to the quantization of the electric charge even without the consideration of magnetic monopoles as suggested in a classical paper of Dirac[3].

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1. W. Israel, *Nuovo Cimento* **44B** (1966) 1;**48B** (1967) 463.
2. V. Berezin, *Phys. Rev.* **D**55 (1997) 2139.
3. P.A.M. Dirac, *Proc. Roy. Soc.* **A**133 (1931) 60.